1. Introduction
   a. Static: Even-odd staggering of Q value
   b. Dynamic: Pair breaking from saddle to scission, at scission
2. Thermodynamics, grand-canonical
   a. Boltzmann gas (Wilkins and Steinberg)
   b. Shifted Fermi gas (BCS) (Fong, Asghar)
3. Other statistical approaches
   a. Combinatory of pairs (Nifenecker)
   b. Statistics of quasi-particles (Mantzouranis and Nix)
4. Excited states of a finite Fermionic system with fixed energy
   a. Analytical model of Strutinsky
   b. Super-fluid nuclear model of Ignatyuk
5. Confrontation with observations
Static and dynamical aspects of the even-odd effect in fission

Case: Even-Z nucleus passes the fission barrier fully paired.
Even and odd charge splits differ in Q value by $\Delta$.
Odd-Z fragments can only be produced by quasi-particle excitations before or at scission.

Controversy:
Can the statistical model “explain” the even-odd effect in the yields?
- Quasi-particle excitations on the way from saddle to scission.
- Yields of different fragments due to the number of available states.

Alternative option: Pair breaking during neck rupture.
Thermodynamical approach

Ideal gas:

\[ y \propto \exp\left( -\frac{E}{T} \right) \]

(Boltzmann)

Applied e.g. by Wilkins and Steinberg, PLB42 (1972) 141, in their scission-point model.

Not appropriate for finite super-fluid Fermionic system!
Shifted Fermi-gas model

Boltzmann replaced by Fermi-gas level density

Asymptotic behaviour modelled by shift of $\Delta$ for even-odd and $2\Delta$ for even-even nuclei.

Even-odd fluctuations in Q value are exactly balanced by shift of level density (e.g. Medkour et al., JPG 23 (1997) 103). No even-odd effect in yields expected!
Other statistical approaches

Nifenecker et al. ZPA308 (1982) 39:
Statistical combinatorial approach based on the number of broken pairs.

Mantzouranis and Nix, PRC25 (1982) 918:
Statistical approach based on the ratio of the number of quasiparticle excitations.

Both approaches are not consistent with the basics of statistical model: The number of available states.
Excited states of a super-fluid Fermionic system

1. Analytical approximation of Strutinsky (1958)
   - Fixed energy (not temperature!)
   - Super-fluid system
   - Restriction to one component

   \[
   \rho_n(U) = \frac{g^n(U - n\Delta)^{n-1}}{\left[(n/2)!\right]^2 (n-1)!}
   \]

   **Approximation:** \(\Delta\) does not depend on energy. (This is not critical for the lowest excitation energies considered here)

Relation to shifted Fermi-gas model:
   1. Deviations for lowest energies appear!
   2. Extension to two-component system required!
Excited states of a super-fluid Fermionic system

2. Analytical description of Ignatyuk et al. SJNP17 (1973) 376

- Extension to two-component system
- Inclusion of Pauli blocking
- Variation of pairing strength

Stringent formulation of the number of excited states in the proton and neutron subsystem with N quasiparticles.

Neutron and proton excitations with different numbers of quasiparticles are in competition!
Excited states of a super-fluid Fermionic system

3. Application to fission by Rejmund et al. NPA678 (2000) 215

Probability for fully paired proton configuration:

$$P_0^Z (U) = \frac{\sum_{n_N} \rho_{n_Z=0,n_N} (U)}{\sum_{n_Z,n_N} \rho_{n_Z,n_N} (U)}$$

($P_0^Z = \text{observed even-odd effect}$)

Finite probability for $U > \Delta$ that protons (neutrons) remain completely paired.

Purely statistical reasoning can explain the even-odd effect in fission!
Observations

1. Magnitude of proton and neutron even-odd effects,
2. Energy dependence

Explanation of drastic difference between proton and neutron even-odd effect.

“Reasonable” excitation-energies at scission.

Data for high TKE: no neutron evaporation possible.
Deduced intrinsic excitation energy at scission

$\delta_Z$: Fraction of proton QP excitations at scission

Rejmund et al. NPA678 (2000) 215
Nifenecker et al. ZPA308 (1982) 39

$\Delta V$: Potential energy gain (saddle-scission)

$E_{\text{diss}}$: Deduced dissipated energy

$E_{\text{diss}}/\Delta V$: 30% to 40%
Observations

3. Variation with asymmetry (Steinhäuser et al. NPA634 (1998) 89)

Even-odd effect in mass-asymmetric splits (also for odd-Z nuclei!) due to larger single-particle level density in larger fragment.
Statistical considerations predict even-odd effect in fission.

Rigorous formulation of the level density is essential.

Many features of experimental data are described:
  Amplitude in neutron and proton number,
  Decrease with excitation energy,
  Increase with mass asymmetry.

This success revitalizes the discussion on dynamical or statistical interpretation of the fission process.