Possible Spectrometer for eA Collider

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• Basic idea
• Magnetic field
• Expected performances
• Size of dipole magnet
• Merits and demerits
• Spectrometer without the front counter
• Conclusion
Requirements to the collider spectrometer

<table>
<thead>
<tr>
<th>Requirement</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>momentum resolution</td>
<td>$10^{-4}$</td>
</tr>
<tr>
<td>angular resolution</td>
<td>1 mr</td>
</tr>
<tr>
<td>solid angle</td>
<td>40 msrc</td>
</tr>
<tr>
<td>maximum momentum</td>
<td>800 MeV/c</td>
</tr>
<tr>
<td>momentum range</td>
<td>10 %</td>
</tr>
<tr>
<td>minimum angle</td>
<td>6°</td>
</tr>
<tr>
<td>colliding length acceptance</td>
<td>10 cm</td>
</tr>
</tbody>
</table>
A double-arm, large acceptance spectrometr “YOKAN” discussed in the collaboration of SMART spectrometer.

\[ \text{beam} \]

\[
\begin{array}{c|c|c}
\text{50 MeV} & \text{200 MeV} \\
\hline
5^\circ & 39.7^\circ & 6.17^\circ \\
15^\circ & 14.21^\circ & 4.36 \\
25^\circ & 8.91^\circ & 3.65 \\
35^\circ & 4.12 & 1.73 \\
45^\circ & 3.26 & 1.61 \\
55^\circ & 2.99 & 4.59 \\
65^\circ & 2.64 & 4.42 \\
\end{array}
\]

\[ \text{gap} = 20 \text{ cm} \quad (200 \text{ MeV}, t \ldots 48.1 \text{ mrad}, 5\sim 65^\circ) \]

\[ \Delta \omega = 159.45 \text{ mrad} = 48.7 \text{ mrad} \]
Possible spectrometers which do not deflect nor stop the beam

1. Conventional spectrometers at the nearest approach to the beam
   MAMI-C(6°), JLAB(12°-->6° with septums)

2. Spectrometers in which beams go straight due to special conditions.
   2-1 Uniform solenoid field
       along the field line ---\> $B \parallel p$
       conventional collider spectrometers

   2-2 Quadrupole field
       along the symmetric axis ---\> $B = 0$
       “YOKAN” spectrometer
Basic idea of the Q-magnet-based spectrometer

- Electron and RI beams collide each other along the symmetry axis of the quadrupole magnet of the spectrometer.
- Intact beams go straight along the field-free, symmetrical axis of the quadrupole magnet.
- Scattered electron are focused vertically, magnifying the acceptance.
- They are horizontally defocused, magnifying the angle of exit.
- Electrons are extracted from the side face of the quadrupole magnet.
Electrons scattered to extreme forward angles can be analyzed.

They are then analyzed by a dipole magnet.

The exit angle from the quadrupole magnet is almost constant.

(demerit) We lose significant part of the information on scattering angles.

(merit) The scattering angle can be changed without rotating the dipole magnet if we adjust the strength of the quadrupole magnet and/or the colliding position of the beams.
Possibilities to change the detection angle

1. Move the dipole magnet parallel to the beam line
   --- It may be easier than the rotation.

2. Adjust the collision point and Q-magnet strength

3. Adjust only the Q-magnet strength

4. Enlarge the horizontal angular acceptance
Change the detection angle by beam and Q-magnet

Strength of the Q-magnet and the colliding position are adjusted
Extreme forward and extreme backward angles

20 ~ 100 mr

780 ~ 880 mr
reversed polarity
Change the detection angle only by Q-magnet

merit: fixed collision position
demerit: reduced horizontal acceptance

100~300 mr

300~500 mr
Cover the necessary angular range with one shot
Resolutions

Counter resolution 0.2 mm
Multiple scattering 0.5 mr

Counter resolution 0.1 mm
Multiple scattering 0.2 mr

![Graph 1](image1.png)

![Graph 2](image2.png)
Dependence on the colliding length
Why the colliding length acceptance is so large

It is because (x|y) is very small at the focal plane (y means the source position along the beam direction).

\[ \delta y = 50 \text{ cm} \]
\[ \delta \theta = 0 \]

\[ \delta y = 0 \]
\[ \delta \theta = 200 \text{ mr} \]
From traditional to precise expression of fringing field

Traditional method

(J.E.Spencer & H.A.Engel: NIM 49(1967)181)

Define $s = x / G$ and fit $B_y$ along x-axis by

$$h(s) = \frac{1}{1 + \exp(S)}$$

$$S = c_0 + c_1 s + c_2 s^2 + c_3 s^3 + c_4 s^4 + c_5 s^5$$

Extension to two-dimensional space:

$$B_y(x, y) = B_y(x, 0) + \frac{y^2}{2!} \frac{\partial^2 B_y}{\partial y^2} + ....$$

$$= B_0 [h(x) - \frac{y^2}{2!} \frac{d^2 h}{dx^2} + ....]$$

$$B_x(x, y) = B_0 [y \frac{dh}{dx} - \frac{y^3}{3!} \frac{d^3 h}{dx^3} + ....]$$

$$\nabla^2 \mathbf{B} = 0 \text{ if } \mathbf{j} = 0$$
Field strength distribution

From the field calculation

From the parameters
up to 2-nd order

We need precise value up to $y = G/2$.

There is no reason to justify the cut-off at the 2-nd order term.

Can we sum up to infinity?
From the parameters
exact summation

Occurrence of
singularity cannot be avoided.
Exact summation up to infinity.

If the field is independent of the \( z \)-coordinate

\[
B_y(x,y) = B_y(x,0) + \frac{y^2}{2!} \frac{\partial^2 B_y}{\partial y^2} + \frac{y^4}{4!} \frac{\partial^4 B_y}{\partial y^4} + \frac{y^6}{6!} \frac{\partial^6 B_y}{\partial y^6} + \ldots
\]

\[
= B_0 \left[ h(x) - \frac{y^2}{2!} \frac{d^2 h}{dx^2} + \frac{y^4}{4!} \frac{d^4 h}{dx^4} - \frac{y^6}{6!} \frac{d^6 h}{dx^6} + \ldots \right]
\]

\[
= B_0 \cos( y \frac{d}{dx}) h(x) = B_0 \text{Re}[\exp( iy \frac{\partial}{\partial x}) h(x)]
\]

\[
= B_0 \text{Re}[ h(x + iy)]
\]

\[
B_x(x, y) = B_0 \text{Im}[ h(x + iy)]
\]

There is no reason to terminate the summation at a finite order when the exact summation is possible.
Occurrence of singularities in two-dimensional space

\[ h(s) = \frac{1}{1 + \exp(S)} \]
\[ S = c_0 + c_1 s + c_2 s^2 + c_3 s^3 + c_4 s^4 + c_5 s^5 \]
\[ s = \frac{x}{G} + i \frac{y}{G} \]

For complex \( S \), \( \exp(S) = -1 \) is possible.

We have to know the location of singularities by solving following equation:

\[ c_0 + c_1 s + c_2 s^2 + c_3 s^3 + c_4 s^4 + c_5 s^5 = (2m + 1)\pi i \]

(generally unsolvable)
Enge’s long-tail parameter set gives strange field

Up to 2-nd order

Up to infinite order
A new fitting function

\[ h(s) = \frac{1}{[1 + \exp\left(\frac{s-b}{a}\right)]^n} \]

\[ s = \frac{x}{G} + i \frac{y}{G} \]

location of singularities

\[ x = bG \]
\[ y = \pi a G, \ 3\pi a G, \ 5\pi a G, \ldots \]

safety condition

\[ b < 0 \]
\[ a > \frac{1}{2\pi} \approx 0.16 \]
Enge’s short tail parameter set gives ....

original parameters

parameters converted
Field of the quadrupole magnet

Parameters were extracted by fitting
\[ h(s) = (1 + e) \frac{1 + \frac{1}{2} [cs - 1 - \sqrt{(cs - 1)^2 + d}]}{[1 + \exp\left(\frac{s - b}{a}\right)]^n} \]
to \( B_y(x,0) \) along \( x \)-axis with a variable
\[ s = \frac{x}{G}. \]

The fitting can be extended to full space by defining
\[ s = \frac{x}{G} + i \frac{y}{G} \]

\[ B_x(x, y) = B_0 \text{Im}[h(s)] \]
\[ B_y(x, y) = B_0 \text{Re}[h(s)] \]
standard 20 msr version
slim 10 msr version
Pair spectrometer 1

same polarity
Pair spectrometer 2
opposite polarity
## Specification of the magnets

<table>
<thead>
<tr>
<th>Quadrupole Magnet</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Gap of quadrupoles</td>
<td>32 cm</td>
</tr>
<tr>
<td>Gap of dipoles</td>
<td>10 cm</td>
</tr>
<tr>
<td>Field gradient</td>
<td>6 T/m</td>
</tr>
<tr>
<td>Maximum field</td>
<td>1.5 T</td>
</tr>
<tr>
<td>Mass</td>
<td>30 ton</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Dipole Magnet</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum field</td>
<td>1.4 T</td>
</tr>
<tr>
<td>Gap</td>
<td>24 cm</td>
</tr>
<tr>
<td>Mass</td>
<td>280 ~ 540 ton</td>
</tr>
<tr>
<td>Power consumption</td>
<td>520 ~ 410 kW</td>
</tr>
</tbody>
</table>
Comparison with existing spectrometers

<table>
<thead>
<tr>
<th>type</th>
<th>conventional</th>
<th>cylindrical</th>
<th>Q-magnet based</th>
</tr>
</thead>
<tbody>
<tr>
<td>example</td>
<td>MAMI-B</td>
<td>OPAL</td>
<td>present</td>
</tr>
<tr>
<td>target</td>
<td>fixed</td>
<td>beam</td>
<td>beam</td>
</tr>
<tr>
<td>mom. resolution</td>
<td>high</td>
<td>low</td>
<td>high</td>
</tr>
<tr>
<td>colliding length</td>
<td>5 cm</td>
<td>~ 1 m</td>
<td>10 cm</td>
</tr>
<tr>
<td>acceptance</td>
<td>(dipole gap)</td>
<td>(vertex counter)</td>
<td></td>
</tr>
<tr>
<td>magnet</td>
<td>D</td>
<td>solenoid</td>
<td>QD</td>
</tr>
<tr>
<td>focal plane</td>
<td>exists</td>
<td>do not exist</td>
<td>exists</td>
</tr>
<tr>
<td>symmetry</td>
<td>vertical plane</td>
<td>beam axis</td>
<td>horizontal plane</td>
</tr>
<tr>
<td>solid angle</td>
<td>5.6 msr</td>
<td>~ 4\pi sr</td>
<td>10~20 msr</td>
</tr>
<tr>
<td>minimum angle</td>
<td>7°</td>
<td>?</td>
<td>1°</td>
</tr>
</tbody>
</table>
Merits and demerits of the present spectrometer

merits
• high resolution
• extreme forward angle
• large acceptance of colliding length
• no need of the rotation
• existence of focal plane
• horizontal median plane
• simple structure

demerits
• interference with the beam
• poor resolution of horizontal angle
• angle dependence of focusing property
• no defining slit
Can we improve the horizontal angular resolution by removing the front counter which causes the serious multiple scattering?

\[ \phi < \pm 80 \text{ mr} \]

- removal of the front counter
- decrease of momentum resolution
- necessity of dispersion increase
- increase of bending angle
- decrease of horizontal acceptance
- necessity of increasing the vertical angle acceptance
- large gap of quadrupole magnet

\[ 100 \text{ mr} < \theta < 200 \text{ mr} \]
Dependence of resolutions on the beam length and on the vertical angle

![Graph showing the dependence of resolutions on the beam length and vertical angle.](image)
### Required and possible performances

<table>
<thead>
<tr>
<th></th>
<th>required</th>
<th>With front counter</th>
<th>Without front counter</th>
</tr>
</thead>
<tbody>
<tr>
<td>momentum resolution</td>
<td>$10^{-4}$</td>
<td>$(0.5\sim1) \times 10^{-4}$</td>
<td>$1 \times 10^{-4}$</td>
</tr>
<tr>
<td>angular resolution</td>
<td>1 mr</td>
<td>1 ~ 4 mr</td>
<td>1 mr</td>
</tr>
<tr>
<td>solid angle</td>
<td>40 msr</td>
<td>10 ~ 40 msr</td>
<td>12 msr</td>
</tr>
<tr>
<td>maximum momentum</td>
<td>800 MeV/c</td>
<td>800 MeV/c</td>
<td>800 MeV/c</td>
</tr>
<tr>
<td>momentum range</td>
<td>10 %</td>
<td>10 %</td>
<td>10 %</td>
</tr>
<tr>
<td>minimum angle</td>
<td>6 °</td>
<td>1 °</td>
<td>6 °</td>
</tr>
<tr>
<td>maximum angle</td>
<td>30 °</td>
<td>50 °</td>
<td>23 ° ?</td>
</tr>
<tr>
<td>colliding length acceptance</td>
<td>10 cm</td>
<td>10 ~ 50 cm</td>
<td>&lt; 10 cm</td>
</tr>
</tbody>
</table>
Conclusion

The spectrometer equipped with the front counter has many advantages.

- The momentum resolution is enough.
- More than enough angular range can be accepted without rotation.
- More than enough colliding length can be accepted.
- The structure is very simple.

The most serious disadvantage is the poor angular resolution.

It can be improved by removing the front counter and by losing some of the advantages.

The choice depends on the counter development, the beam development, and the requirement of the physics.
Ray-tracing calculation for designing a magnetic spectrometer

1. Prepare magnetic field distribution.
   It has to be as realistic as possible.

2. Trace rays by solving the equation of motion
   \[ m \frac{d^2 \mathbf{r}}{dt^2} = q \mathbf{v} \times \mathbf{B} \]
   with appropriate initial conditions.

3. Evaluate the optical properties at the counter position.
   image sharpness, momentum sensitivity, etc.
Field line and field strength distributions of BBS-Q1 magnet
Momentum resolution of a spectrometer

resolution = \frac{\text{separation at counter}}{\text{beam image size at counter}}

= \frac{\text{dispersion}}{(\text{beam size})(\text{magnification}) + (\text{aberration})}

aberration elimination: hardware correction vs software correction
**dipole magnet**

<table>
<thead>
<tr>
<th>uniform field width (m)</th>
<th>H-type</th>
<th></th>
<th>window-frame</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>mass (ton)</td>
<td>power (kW)</td>
<td>mass (ton)</td>
<td>power (kW)</td>
</tr>
<tr>
<td>1.0</td>
<td>320</td>
<td>350</td>
<td>130</td>
<td>440</td>
</tr>
<tr>
<td>1.2</td>
<td>370</td>
<td>370</td>
<td>160</td>
<td>460</td>
</tr>
<tr>
<td>1.4</td>
<td>420</td>
<td>380</td>
<td>200</td>
<td>480</td>
</tr>
<tr>
<td>1.6</td>
<td>480</td>
<td>390</td>
<td>240</td>
<td>500</td>
</tr>
<tr>
<td>1.8</td>
<td>540</td>
<td>410</td>
<td>280</td>
<td>520</td>
</tr>
<tr>
<td>2.0</td>
<td>600</td>
<td>420</td>
<td>330</td>
<td>530</td>
</tr>
<tr>
<td>2.2</td>
<td>670</td>
<td>440</td>
<td>380</td>
<td>550</td>
</tr>
<tr>
<td>2.4</td>
<td>740</td>
<td>450</td>
<td>430</td>
<td>570</td>
</tr>
<tr>
<td>2.6</td>
<td>810</td>
<td>460</td>
<td>480</td>
<td>590</td>
</tr>
</tbody>
</table>

\[ B = 1.4 \text{ T}, \  L = 2.7 \text{ m}, \  G = 25 \text{ cm} \]
Resolutions of the fat version